

Title: “**Disjunctive sequences, discontinuous systems and fast basins**”

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Let I be a finite alphabet. A sequence $(i_n)_{n=1}^\infty$ is said to be *disjunctive* provided every finite word $\tau_1 \dots \tau_L \in I^L$ appears (infinitely often) in $(i_n)_{n=1}^\infty$: $\tau_1 \dots \tau_L = \sigma_{n+1} \dots \sigma_{n+L}$ for some n .

Disjunctive sequences are combinatorially rich. Hence they are studied in algorithmic complexity where one considers them to be random although deterministic. Since a disjunctive sequence cannot be almost periodic, we can also classify it as chaotic. The subset of the coding space I^∞ comprising all nondisjunctive sequences is small in both the probabilistic and Baire category sense.

Around 1991 G.S. Goodman observed that the canonical explanation of the chaos game for hyperbolic IFSs by Elton’s ergodic theorem can be substituted by merely exhausting all addresses on the attractor. Recent advances in the study of random iteration for nonhyperbolic IFSs show that, in general, we can expect the following phenomenon. Given an IFS $F = (X; f_i : X \rightarrow X, i \in I)$ and a minimal F -invariant compact set $\emptyset \neq C = \overline{\bigcup_{i \in I} f_i(C)} \subset X$, we can recover C , simply by running the iteration $x_n = f_{i_n}(x_{n-1})$ driven by a disjunctive sequence $(i_n)_{n=1}^\infty \in I^\infty$. Indeed, this holds true when C is a strict attractor and additionally, either f_i ’s are nonexpansive, or C has a strongly-fibred structure. Interestingly, the algorithm works for some discontinuous IFSs (relating to beta-transformations) which lack a strict attractor. These phenomena could have find applications in control and optimization.

Finally, we would like to point out that, unexpectedly, the existence of attractors for some IFSs comprising discontinuous maps can be related to fast basins. One should note that the reason to introduce fast basins comes from the fractal extension of the analytic continuation theory.

References

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