Title: "**Dynamics of some models from the economics curriculum: interactive activity with GeoGebra**" Speaker: Krzysztof Leśniak (Nicolaus Copernicus University, Toruń, Poland)

We would like to show how simple and more complicated applets, build in GeoGebra [1], can facilitate playful understanding of the dynamics behind "statically looking" mathematical formulas. The examples include the Hotelling long street [2] and the comparison between the Cournot and Stackelberg duopolies, see Fig.1.

We also hope to present some other examples from the dynamics, e.g., bifurcations in oligopolies according to [4], see Fig.2.

Let us briefly recall what is a symmetric oligopoly with quadratic payoffs and what dynamics we can impose on it, cf. [3,4]. Fix N firms, each manufacturing the same product (at least in consumer's view). In the case of $N = 2$ we speak about duopoly. The Hotelling street is a duopoly with special nonquadratic payoffs (defined geometrically). Denote by $q_i \in [0, L]$ the level of production of the i-th firm, $i = 1, .., N$, under the capacity of production L. Every firm taking part in the concurrency game receives payoff

$$P_i(q_1, ..., q_N) = q_i \cdot (a - b \cdot Q) - c \cdot q_i,$$

dependent on the total supply $Q = \sum_{j=1}^{N} q_j$, the unit price a assuming no competition, the market saturation b and the unit cost of production c.

One possible dynamics associated with oligopoly is to consider the following simple strategy: in the next stage of the game each player uses his best reply $R(Q)$ to the previously observed level of total production Q, i.e., $q_i' = R(Q)$, where

$$R(Q) = arg \max_{q_i} \; P_i(q_i, Q).$$

A more refined approach is to suppose that firms cannot immediately adjust their productions to the best response level. Hence their response locates somewhere inbetween the best reply and the continuation of the previous strategy, say

$$q_i' = v \cdot R(Q) + (1 - v) \cdot q_i.$$

Thus we have a kind of a speed of adjustment v. In the case of maximal speed $v = 1$ we recover the best reply dynamics.

References

[1] Geogebra, http://www.geogebra.org

[2] *Hotelling's law*, http://geogebratube.org/student/m542

[3] J. Watson, *Strategy: An Introduction to Game Theory*, W.W. Norton 2002

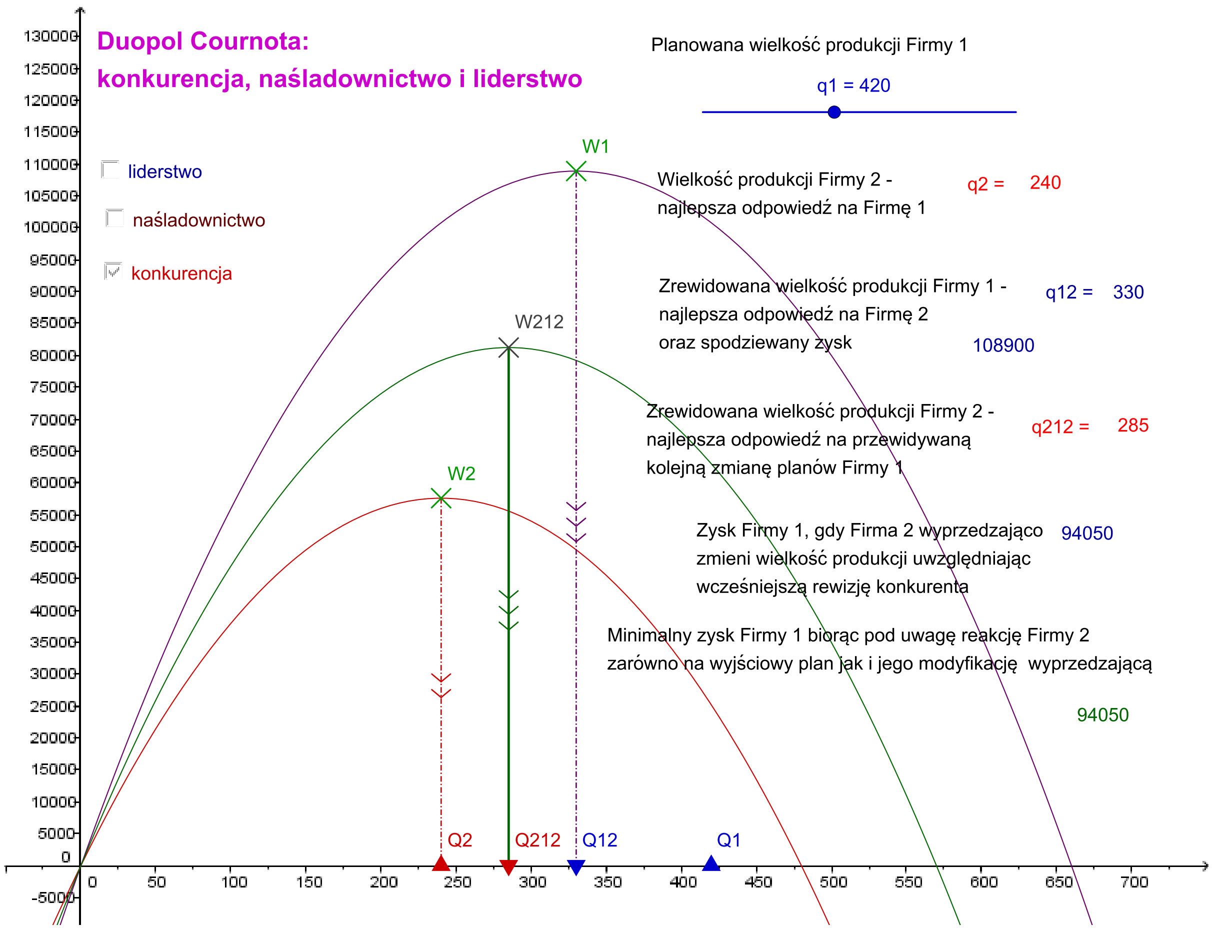

Figure 1: Cournot and Stackelberg duopolies; visualization of a discussion in [3].

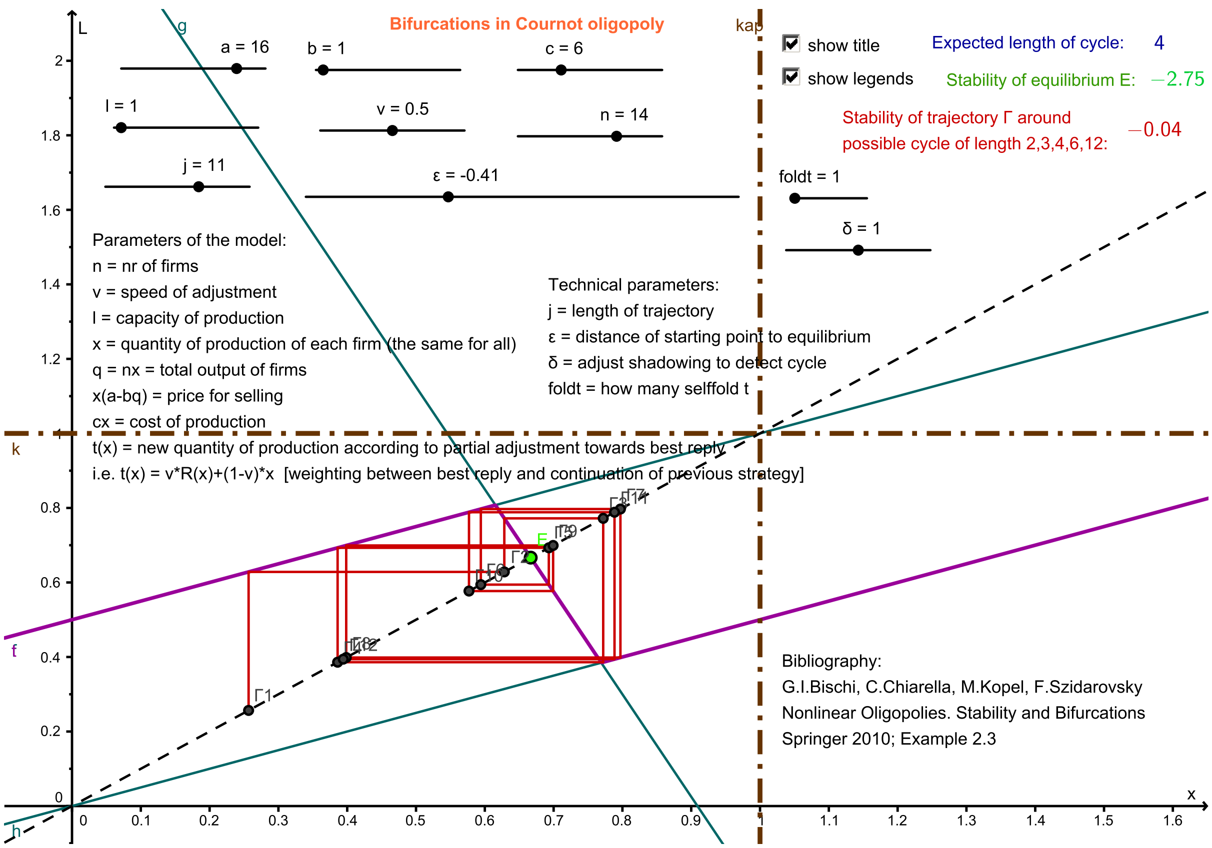

Figure 2: Detecting attractive orbits in a symmetric Cournot oligopoly for 14 firms; visualization of Example 2.3 in [4].

[4] G.I. Bischi, C. Chiarella, M. Kopel, F. Szidarovszky, *Nonlinear Oligopolies: Stability and Bifurcations*, Springer 2010