RAY CATEGORIES, III

BASED ON THE TALK BY DIETER VOSSIECK

DEFINITION.

By a zigzag in a ray category X we mean every sequence $(\alpha_i)_{i\in I}$ of morphisms in the category X, where either $I = [1, n]$ for some $n \in \mathbb{N}$ or $I = \mathbb{N}_+$, such that the following conditions are satisfied:

(1) if $I = [1, 2]$ then either $s\alpha_1 = s\alpha_2$ or $t\alpha_1 = t\alpha_2$,
(2) for each $i \in I$ such that $i - 1, i + 1 \in I$ either $s\alpha_i = s\alpha_{i-1}$ and $t\alpha_i = t\alpha_{i+1}$ or either $t\alpha_i = t\alpha_{i-1}$ and $s\alpha_i = s\alpha_{i+1}$,
(3) for each $i \in I$ such that $i + 1 \in I$ and $s\alpha_i = s\alpha_{i+1}$ there is no morphism β in the category X such that either $\alpha_i = \beta\alpha_{i+1}$ or $\alpha_{i+1} = \beta\alpha_i$,
(4) for each $i \in I$ such that $i + 1 \in I$ and $t\alpha_i = t\alpha_{i+1}$ there is no morphism β in the category X such that either $\alpha_i = \alpha_{i+1}\beta$ or $\alpha_{i+1} = \alpha_i\beta$.

DEFINITION.

We say that a ray category X is zigzag finite if there are no infinite zigzags starting in the category X.

THEOREM (FISCHBACHER).

If X is a zigzag finite connected ray category, then

(1) the group is $\pi_1(X)$ is free,
(2) $H^2(X, G) = 0$ for each group G,
(3) the universal cover $\tilde{X}$ is interval finite.

THEOREM (BONGARTZ).

A connected ray category X is locally representation finite if and only if its universal cover $\tilde{X}$ is zigzag finite and does not contain a convex subcategory which is tame concealed.

Date: 17.07.2009.