# MORE ON FINITE COMPLEXITY

BASED ON THE TALK BY DAN ZACHARIA

Throughout the talk $\Lambda$ is a fixed selfinjective algebra.

Definition.

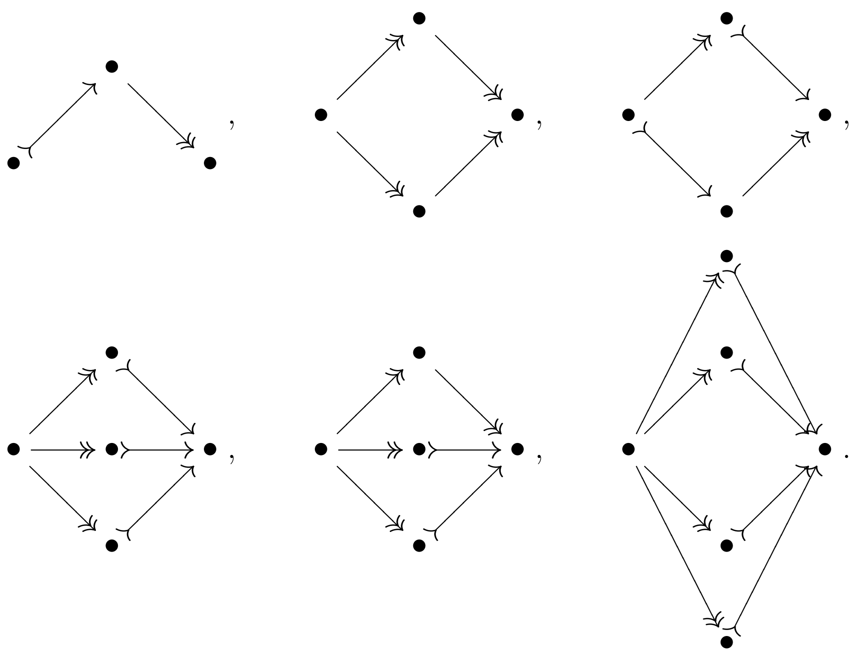
An indecomposable $\Lambda$-module $C$ is called $\Omega$-*perfect* if the following conditions are satisfied:

(1) for each irreducible map $g$ terminating at $C$ either $\Omega^n g$ is an epimorphism for each $n \in \mathbb{N}$ or $\Omega^n g$ is a monomorphism for each $n \in \mathbb{N}$,

(2) for each irreducible map $g$ starting at $C$ either $\Omega^n g$ is an epimorphism for each $n \in \mathbb{N}$ or $\Omega^n g$ is a monomorphism for each $n \in \mathbb{N}$.

Theorem (Green/Zacharia).

If a module $C$ is $\Omega$-perfect and $\operatorname{cx} C < \infty$, then the Auslander–Reiten sequence terminating at $C$ has one of the following forms:



Theorem (Green/Zacharia).

If there are no periodic simple $\Lambda$-modules, then every indecomposable $\Lambda$-module is eventually $\Omega$-periodic.

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*Date*: 10.07.2009.

Theorem (Kerner/Zacharia).

Let $\mathscr{C}$ be a component of the Auslander–Reiten quiver of the algebra $\Lambda$ such that every module in the quiver $\mathscr{C}^s$ is eventually $\Omega$-periodic and no module in the quiver $\mathscr{C}^s$ is $\tau$-periodic. If the quiver $\mathscr{C}^s$ contains a module of finite complexity, the $\mathscr{C}^s = \mathbb{Z}\Delta$, where the quiver $\Delta$ is either extended Dynkin or infinite Dynkin.