RAY CATEGORIES, I

BASED ON THE TALK BY DIETER VOSSIECK

Definition.

By a *ray category* we mean every category X such that the following conditions are satisfied:

(1) The objects of the category X form a set and are pairwise nonisomorphic.
(2) There exists a family $0_{x,y} : x \to y$, $x, y \in X$, of zero morphisms, i.e. $0 \circ \mu = 0 = \nu \circ 0$.
(3) For each object x of the category X the sets $\bigcup_{y \in X} X(x, y)$ and $\bigcup_{y \in X} X(y, x)$ are finite.
(4) For each object x of the category X there exists an endomorphism ρ_x and a positive integer n such that
$$X(x, x) = \{\mathbf{1}_x, \rho_x, \dots, \rho_x^{n-1}, 0\}.$$
(5) For each pair (x, y) of objects of the category X there exists a morphism $\mu \in X(x, y)$ and a positive integer m such that either
$$X(x, y) = \{\mu, \mu\rho_x, \cdots, \mu\rho_x^{m-1}, 0\}$$
or
$$X(x, y) = \{\mu, \rho_y\mu, \cdots, \rho_y^{m-1}\mu, 0\}.$$
(6) If μ, μ', ν, ν' are morphisms in the category X, then the following conditions are satisfied:
$$\text{if } 0 \neq \mu\nu = \mu\nu', \text{ then } \nu = \nu',$$
and
$$\text{if } 0 \neq \mu\nu = \mu'\nu, \text{ then } \mu = \mu'.$$

Remark.

The last condition in the above definition is equivalent to the following condition: if $\nu\xi\mu = \nu\mu$ for morphisms ν, ξ, and μ in the category X, then $\xi = \mathbf{1}$.

Definition.

A morphism μ in a ray category X is called *irreducible* if $\mu \neq 0$, $\mu \neq \mathbf{1}$, and μ has no proper factorization.

Remark.

Any morphism in a ray category is a composition of irreducible ones.

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Notation.

Given a ray category X we denote by QX its quiver defined as follows: the vertices of the quiver QX are the objects of the category X and the arrows of the quiver QX are the irreducible morphisms in the category X.

Notation.

Given a quiver Q we denote by PQ the path category of Q defined as follows: the objects of the category PQ are the vertices of the quiver Q and the morphisms in the category PQ are given by the paths in the quiver Q plus the formal zero morphisms.

Remark.

For a ray category X we have the canonical full and dense functor $PQX \to X$.

Notation.

Let X be a ray category. For a morphism u in the category PQX we denote by $\overline{u}$ its image under the canonical functor $PQX \to X$.

Notation.

For a ray category X we denote by RX the kernel of the canonical functor $PQX \to X$, i.e.

$$RX(x, y) := \{(u, v) \in PQX(x, y) \times PQX(x, y) \mid \overline{u} = \overline{v}\}.$$

Definition.

By a contour in a ray category X we mean every pair (u, v) of paths in the quiver QX such that $\overline{u} = \overline{v} \neq 0$.

Definition.

By a zero path in a ray category X we mean every path u in the quiver QX such that $\overline{u} = 0$.