SOME CLASSES OF 2-CALABI-YAU TILTED ALGEBRAS GIVEN BY QUIVERS WITH POTENTIALS

BASED ON THE TALK BY IDUN REITEN

Assumption.

Throughout the talk k is a fixed algebraically closed field.

Definition.

By a quiver with potential we mean a finite quiver Q with together with a k-linear combination W, called potential, of cyclic paths.

Notation.

For a quiver Q with a potential W we put

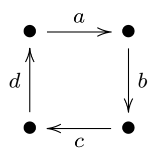
$$\mathscr{P}(Q,W) := k\hat{Q}/\langle \partial_a W \mid a \in Q_1 \rangle.$$

Definition.

By a Jacobi algebra we mean an algebra of the form $\mathscr{P}(Q,W)$ for a quiver Q with a potential W.

Example.

If Q is the quiver



and $W = abcd$, then $\partial_a W = bcd$, $\partial_b W = cda$, $\partial_c W = dab$, and $\partial_d W = abc$.

Definition.

We call a Hom-finite triangulated category $\mathscr{C}$ 2-Calabi-Yau if

$$\mathrm{Ext}^1_{\mathscr{C}}(A,B) \simeq D\,\mathrm{Ext}^1_{\mathscr{C}}(B,A)$$

for all $A, B \in \mathscr{C}$.

Definition.

An object T of a 2-Calabi-Yau category is called cluster tilting if $\mathrm{Ext}^1_{\mathscr{C}}(T,T) = 0$ and $X \in \operatorname{add} T$ for each $X \in \mathscr{C}$ such that $\mathrm{Ext}^1_{\mathscr{C}}(T,X) = 0$.

Definition.

By a 2-Calabi-Yau tilted algebra we mean every algebra of the form $\mathrm{End}_{\mathscr{C}}(T)$ for a cluster tilting object in a 2-Calabi-Yau category $\mathscr{C}$.

Date: 06.05.2008.

Remark.

Every cluster tilted algebra is a 2-Calabi-Yau tilted algebra.

Theorem (Amiot).

Every finite dimensional Jacobi algebra is a 2-Calabi-Yau tilted algebra.

Definition.

For a finite quiver Q without loops we denote by W_Q the Coxeter group associated with Q, i.e. the group generated by s_x, $x \in Q_0$, such that $s_x^2 = 1$ for each $x \in Q_0$, $s_x s_y = s_y s_x$ for all $x, y \in Q_0$ such that there is no arrow connecting x and y, and $s_x s_y s_x = s_y s_x s_y$ for all $x, y \in Q_0$ such that there is exactly one arrow connecting x and y.

Definition.

Let Q be a finite quiver without loops. A sequence $(x_1, \ldots, x_n) \in Q_0^n$ is called reduced if there is no sequence $(y_1, \ldots, y_m) \in Q_0^m$ such that $m < n$ and $s_{y_1} \cdots s_{y_m} = s_{x_1} \cdots c_{x_n}$.

Notation.

For a quiver Q without loops we put

$$\Lambda_Q := k\hat{\overline{Q}}/\langle \sum_{a \in Q_1} aa^* - a^*a \rangle,$$

where $\overline{Q}$ is the double quiver of Q. Moreover, for each $x \in Q_0$ we put

$$I_x := \Lambda_Q \cdot (1 - e_x) \cdot \Lambda_Q.$$

Notation.

Let Q be a finite quiver without loops. For a reduced sequence $\mathbf{x} = (x_1, \ldots, x_n) \in Q_0^n$ we put

$$I_{\mathbf{x}} := I_{x_1} \cdots I_{x_n} \qquad \text{and} \qquad \Lambda_{\mathbf{x}} := \Lambda_Q / I_{\mathbf{x}}.$$

Remark.

If $\mathbf{x} = (x_1, \ldots, x_n) \in Q_0^n$ and $\mathbf{y} = (y_1, \ldots, y_n) \in Q_0^n$ are reduced sequences such that $s_{x_1} \cdots s_{x_n} = s_{y_1} \cdots s_{y_n}$, then $I_{\mathbf{x}} = I_{\mathbf{y}}$ and, consequently, $\Lambda_{\mathbf{x}} = \Lambda_{\mathbf{y}}$.

Remark.

If $\mathbf{x} \in Q_0^n$ is a reduced sequence, then $\underline{\mathrm{Sub}}\Lambda_{\mathbf{x}}$ is a 2-Calabi-Yau category.

Notation.

Let Q be a finite quiver without loops. For a reduced sequence $\mathbf{x} = (x_1, \ldots, x_n) \in Q_0^n$ we put

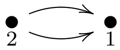
$$T_{\mathbf{x}} := P_{x_1}/I_{x_1}P_{x_1} \oplus \cdots \oplus P_{x_n}/I_{x_1} \cdots I_{x_n} P_{x_n}.$$

Remark.

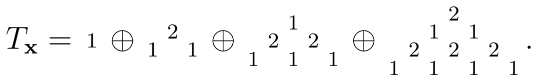
If $\mathbf{x} \in Q_0^n$ is a reduced sequence, then $T_{\mathbf{x}}$ is cluster tilting object in $\underline{\mathrm{Sub}}\Lambda_{\mathbf{x}}$.

Example.

If Q is the quiver

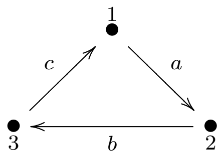


and $\mathbf{x} = (1, 2, 1, 2)$, then

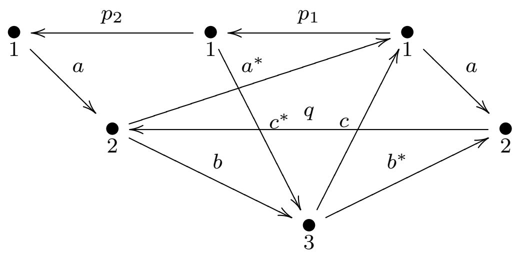


Example.

If Q is the quiver



and $\mathbf{x} = (1, 2, 1, 3, 1, 2, 3, 1, 2)$, then $\mathrm{End}_{\overline{\mathrm{Sub}\Lambda_{\mathbf{x}}}}(T_{\mathbf{x}}) \simeq \mathscr{P}(Q', W)$, where Q' is the quiver



and

$$W := aa^*p_1p_2 - a^*aq + bb^*q - c^*cp_1.$$