MODELING RICHARDSON ORBITS VIA DELTA-FILTERED MODULES FOR AN AUSLANDER ALGEBRA

BASED ON THE TALK BY KARIN BAUR

Assumption.

Throughout the talk k will be a fixed algebraically closed field.

Notation.

For $N \in \mathbb{N}_+$ we put

$$\mathrm{Par}(N) := \{(d_1, \ldots, d_n) \in \mathbb{N}_+^n \mid n \in \mathbb{N}_+ \text{ and } d_1 + \cdots + d_n = N\}.$$

Notation.

Let $N \in \mathbb{N}_+$. For $\mathbf{d} = (d_1, \ldots, d_n) \in \mathrm{Par}(N)$ we put

$$P(\mathbf{d}) := \{g \in \mathrm{SL}_N \mid g_{i,j} = 0 \text{ if } j \leq d_1 + \cdots + d_l < i \text{ for some } l \in [1, n]\}$$

and

$$\mathfrak{n}(\mathbf{d}) := \{X \in \mathfrak{sl}_N \mid X_{i,j} = 0 \text{ if } i > d_1 + \cdots + d_{l-1} \text{ and } j \leq d_1 + \cdots + d_l \text{ for some } l \in [1, n]\}.$$

Remark.

If $N \in \mathbb{N}_+$ and $\mathbf{d} \in \mathrm{Par}(N)$, then $P(\mathbf{d})$ acts on $\mathfrak{n}(\mathbf{d})$ by conjugation.

Theorem (Richardson).

If $N \in \mathbb{N}_+$ and $\mathbf{d} \in \mathrm{Par}(N)$, then $P(\mathbf{d})$ possesses an open and dense orbit in $\mathfrak{n}(\mathbf{d})$.

Theorem (Hille–Röhrle).

If $N \in \mathbb{N}_+$ and $(d_1, \ldots, d_n) \in \mathrm{Par}(N)$, then there is only a finite number of $P(\mathbf{d})$-orbits in $\mathfrak{n}(\mathbf{d})$ if and only if $n \leq 5$.

Notation.

For $n \in \mathbb{N}_+$ let $\mathscr{A}_n$ be the path algebra of the quiver

$$\begin{tikzcd} \underset{1}{\bullet} \arrow[r, bend left, "\alpha_1"] & \underset{2}{\bullet} \arrow[l, bend left, "\beta_1"] \arrow[r, bend left, "\alpha_2"] & \cdots \arrow[l, bend left, "\beta_2"] \arrow[r, bend left, "\alpha_{n-2}"] & \underset{n-1}{\bullet} \arrow[l, bend left, "\beta_{n-2}"] \arrow[r, bend left, "\alpha_{n-1}"] & \underset{n}{\bullet} \arrow[l, bend left, "\beta_{n-1}"] \end{tikzcd}$$

bound by relations $\alpha_i\beta_i - \beta_{i+1}\alpha_{i+1}$, $i \in [1, n-2]$, and $\alpha_{n-1}\beta_{n-1}$.

Notation.

For $n \in \mathbb{N}_+$ and $i \in [1, n]$ we put

$$\Delta(i) := P_{\mathscr{A}_n}(i)/P_{\mathscr{A}_n}(i+1),$$

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where $P_{\mathscr{A}_n}(n+1) := 0$.

NOTATION.

For $n \in \mathbb{N}_+$ let $\mathscr{F}(\Delta)$ be the category of $\mathscr{A}_n$-modules M such that there exists a filtration

$$0 = M_0 \subset M_1 \subset \cdots \subset M_{l-1} \subset M_l = M$$

such that for each $i \in [1, l]$ there exists $j \in [1, n]$ with $M_i / M_{i-1} \simeq \Delta(j)$.

THEOREM (HILLE–RÖHRLE).

Let $N \in \mathbb{N}_+$ and $\mathbf{d} = (d_1, \ldots, d_n) \in \operatorname{Par}(N)$. There exists a bijection between the $P(\mathbf{d})$-orbits in $\mathfrak{n}(\mathbf{d})$ and the isomorphism classes of $\mathscr{A}_n$-modules M such that $M \in \mathscr{F}(\Delta)$ and $[M : \Delta(i)] = d_i$ for all $i \in [1, n]$.