

(a)

$$f := X^4 + X^3 - 3X^2 - 4X - 1.$$

$$g := X^3 + X^2 - X - 1.$$

$$f = X \cdot g - 2 \cdot r_0, \quad r_0 := X^2 + \frac{3}{2}X + \frac{1}{2}.$$

$$g = (X - \frac{1}{2}) \cdot r_0 - \frac{3}{4} \cdot r_1, \quad r_1 := X + 1.$$

$$r_0 = (X + \frac{1}{2}) \cdot r_1 + 0.$$

$$\gcd(f, g) = X + 1.$$

(c)

$$f := X^5 + 3X^2 - 2X + 2.$$

$$g := X^6 + X^5 + X^4 - 3X^2 + 2X - 6.$$

$$g = (X+1) \cdot f + r_0, \quad r_0 := X^4 - 3X^3 - 4X^2 + 2X - 8.$$

$$f = (X + 3) \cdot r_0 + 13 \cdot r_1, \quad r_1 := X^3 + X^2 + 2.$$

$$r_0 = (X - 4) \cdot r_1 + 0.$$

$$\gcd(f, g) = X^3 + X^2 + 2.$$

(b)

$$f := X^6 + 2X^4 - 4X^3 - 3X^2 + 8X - 5.$$

$$g := X^5 + X^2 - X + 1.$$

$$f = X \cdot g + 2 \cdot r_0, \quad r_0 := X^4 - \frac{5}{2}X^3 - X^2 + \frac{7}{2}X - \frac{5}{2}.$$

$$g = (X + \frac{5}{2}) \cdot r_0 + \frac{29}{4} \cdot r_1, \quad r_1 := X^3 - X + 1.$$

$$r_0 = (X - \frac{5}{2}) \cdot r_1 + 0.$$

$$\gcd(f, g) = X^3 - X + 1.$$

(d)

$$f := X^4 + X^3 - 4X + 5.$$

$$g := 2X^3 - X^2 - 2X + 2.$$

$$f = (\frac{1}{2}X + \frac{3}{4}) \cdot g + \frac{7}{4} \cdot r_0, \quad r_0 := X^2 - 2X + 2.$$

$$g = (2X + 3) \cdot r_0 - 4 \cdot r_1, \quad r_1 := 1.$$

$$r_0 = (X^2 - 2X + 2) \cdot r_1 + 0.$$

$$\gcd(f, g) = 1.$$

(a)

$$f := X^4 + 2X^3 - X^2 - 4X - 2.$$

$$g := X^4 + X^3 - X^2 - 2X - 2.$$

$$f = 1 \cdot g + r_0, \quad r_0 := X^3 - 2X.$$

$$g = (X + 1) \cdot r_0 + r_1, \quad r_1 := X^2 - 2.$$

$$r_0 = X \cdot r_1 + 0.$$

$$\gcd(f, g) = X^2 - 2.$$

$$r_1 = 1 \cdot g + (-X - 1) \cdot r_0.$$

$$r_0 = 1 \cdot f + (-1) \cdot g.$$

$$\begin{aligned} \gcd(f, g) &= r_1 = 1 \cdot g + (-X - 1) \cdot r_0 \\ &= 1 \cdot g + (-X - 1) \cdot (1 \cdot f + (-1) \cdot g) \\ &= (-X - 1) \cdot f + (X + 2) \cdot g. \end{aligned}$$

(b)

$$f := 3X^3 - 2X^2 + X + 2.$$

$$g := X^2 - X + 1.$$

$$f = (3X + 1) \cdot g - r_0, \quad r_0 := X - 1.$$

$$g = X \cdot r_0 + r_1, \quad r_1 := 1.$$

$$r_0 = (X - 1) \cdot r_1 + 0.$$

$$\gcd(f, g) = 1.$$

$$r_1 = 1 \cdot g + (-X) \cdot r_0.$$

$$r_0 = (-1) \cdot f + (3X + 1) \cdot g.$$

$$\begin{aligned} \gcd(f, g) &= r_1 = 1 \cdot g + (-X) \cdot r_0 \\ &= 1 \cdot g + (-X) \cdot ((-1) \cdot f + (3X + 1) \cdot g) \\ &= X \cdot f + (-3X^2 - X + 1) \cdot g. \end{aligned}$$